Final Exam – Part II: 150 points

1. (16 pts) Given the figure shown below with $OS \parallel UR$, find the following:

   A. $OS \quad \frac{5}{x} = \frac{3}{10} \quad \Rightarrow \quad \frac{8x = 50}{8} \quad \Rightarrow \quad x = \frac{25}{4} = 6.25''$

   B. $OU \quad \frac{5}{y} = \frac{4}{y} \quad \Rightarrow \quad \frac{xy = 12}{5} \quad \Rightarrow \quad y = 2.4''$

   C. $\frac{\alpha_{UFR}}{\alpha_{OFS}} \quad \frac{8}{5} = 1.6$

   D. $\frac{\alpha_{UFR}}{\alpha_{OFS}} \quad \left(\frac{8}{5}\right)^2 = \frac{64}{25} \quad = 2.56$

4. (5 pts) Bisect the following line segment. Label the midpoint C.
5. (14 points total) Use $\triangle ABC$ to answer the following to the nearest degree. Given $m\angle B = 43^\circ$.

A. What is the length of $AC$?

\[ b^2 = a^2 + c^2 - 2ac \cos 43^\circ \]
\[ b^2 = 18^2 + 20^2 - 2(18)(20) \cos 43^\circ \]
\[ b^2 = 324 + 400 - 720 \cos 43^\circ \]
\[ b^2 = 724 - 720 \cos 43^\circ \]
\[ b^2 = 197.6253346 \]
\[ b = 14.05 \]

B. What is the measure of $\angle A$?

\[ \frac{\sin A}{a} = \frac{\sin B}{b} \]
\[ \frac{\sin A}{18} = \frac{\sin 43^\circ}{14.05} \]
\[ \sin A = \frac{18 \sin 43^\circ}{14.05} \]
\[ A = \sin^{-1} \left( \frac{18 \sin 43^\circ}{14.05} \right) \]
\[ A = 60.89^\circ = 61^\circ \]

C. What is the measure of $\angle C$?

\[ 180^\circ - (61 + 43^\circ) = 180^\circ - 104^\circ = 76^\circ \]

6. (6 pts) What is the measure of $\angle BGE$ if $\angle AGB = 40^\circ$ and $\angle AGE = 135^\circ$?

\[ m \angle AGB + m \angle AGE = m \angle AGE \]
\[ 40^\circ + m \angle BGE = 135^\circ \]
\[ m \angle BGE = 95^\circ \]

Math 97 Final Exam
7. (12 pts) Given the figure shown below. Determine the measures of \( \angle 1 \) through \( \angle 12 \).

\[
\begin{align*}
\angle 1 &= 75^\circ \quad \angle 7 = 20^\circ \\
\angle 2 &= 27^\circ \quad \angle 8 = 58^\circ \\
\angle 3 &= 20^\circ \quad \angle 9 = 47^\circ \\
\angle 4 &= 58^\circ \quad \angle 10 = 133^\circ \\
\angle 5 &= 75^\circ \quad \angle 11 = 47^\circ \\
\angle 6 &= 27^\circ \quad \angle 12 = 133^\circ
\end{align*}
\]

8. (12 pts) A rectangular garden is surrounded on all sides by a path that is 3 feet wide. The dimensions of the courtyard, including path and garden, are 18 feet by 24 feet. How much fencing material is needed to enclose only the garden?

\[
\begin{align*}
\text{Length} &= 24 - 2(3) \\
&= 18 \text{ ft} \\
\text{Width} &= 18 - 2(3) \\
&= 12 \text{ ft} \\
\text{Perimeter of Garden} &= 2L + 2W \\
&= 2(18) + 2(12) \\
&= 36 + 24 \\
\text{Amount of Fence} &= 60 \text{ ft}
\end{align*}
\]
9. (6 pts) Bisect the following angle. Label the bisector $OD$.

10. (8 pts) Draw a line parallel to $BC$ passing through Point A.
11. (12 pts) Fill in the blanks using information from the following figure.

![Diagram of intersecting lines with labeled segments]

A. \( AB + BC > \underline{AC} \)

B. \( \angle 4 + \angle 3 = 180 \) degrees.

C. \( \angle 4 = \angle 5 + \angle 8. \)

D. \( \angle 7 > \angle 5 \) and \( \angle 7 > \angle 3. \)

12. (3 pts each) Do the following lengths of sides form a triangle? If not, state the reason why.

A. 6, 8, 10
   - Yes
   - Pythagorean Test

B. 9, 6, 17
   - No
   - \( 9 + 6 = 15 \), \( 15 \) must be \( > 17 \)

C. \( x, 2x, 3x \)
   - No
   - \( x + 2x = 3x \) which is \( x \) not \( > x \)
13. (12 pts) Complete the following proof, stating the appropriate reasons justifying each statement. (NOTE: Fill in all the blanks in the Statements and Reasons. Not all the lines need to be used.)

Given: \( \angle A = \angle 1 \) and \( \angle 2 = \angle C \)
Prove: \( AB = CD \)

**Statements**

1. \( \angle A = \angle 1 \) and \( \angle 2 = \angle C \)
2. \( BD = AB \)
3. \( BD \leq CD \)
4. \( AB \leq CD \)

**Reasons**

- \( \angle A \) = \( \angle 1 \) and \( \angle 2 = \angle C \) (Given)
- Sides opposite \( \angle A \) are \( \leq \) sides opposite \( \angle C \) (Substitution Property)

14. (8 pts) Given \( \triangle ABC \).

A. Find the area of the triangle.

\[
A = \frac{1}{2} (LBC)(LKB)
\]

\[
BC = \sqrt{17^2 - 15^2} = \sqrt{289 - 225} = \sqrt{64} = 8
\]

\[
A = \frac{1}{2} (8)(15) = 60 \text{ sq units}
\]

B. If the sides of the triangle are doubled, what happens to its area?.

AREA QUADRUPLES
15. (10 points) Use the circle and secants to answer the following.

What is the measure of \( \angle FEC \)?

\[
(6) \quad m \angle AEB = \frac{1}{2}(130 - 28) = \frac{1}{2}(102) = 51^\circ
\]

\[
(4) \quad m \angle FEC = 180 - 51 = 129^\circ
\]

16. (12 pts) Find the total volume of the figure shown below. Use \( \pi = 3.14 \) when necessary. Round to the nearest hundredth.

\[
V_{\text{cyl}} = \pi r^2 h = \pi (12)^2 (25) = 3600 \pi \text{ in}^3
\]

\[
V_{\text{cone}} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (12)^2 (16) = 768 \pi \text{ in}^3
\]

\[
V_{\text{total}} = V_{\text{cyl}} + V_{\text{cone}} = 3600 \pi + 768 \pi = 4368 \pi \text{ in}^3
\]
17. (8 pts) Find the length marked $x$ in the following figure:

\[
\frac{12}{x} = \frac{15}{10}
\]

\[
15x = 120
\]

\[
\frac{15x}{15} = \frac{120}{15}
\]

\[
x = 8
\]
BONUS (total of 20 extra points)

1. (8 pts) In \( \triangle ABC \), \( BD \perp AC \), \( EF \perp AC \), and \( AB \parallel DE \). \( BD = 36 \), \( AD = 16 \), and \( EF = 24 \).

Find the following:

\[
\frac{DF}{24} = \frac{16}{x} \quad \frac{36x}{36} = \frac{24(11)}{36} \quad x = \frac{32}{3} \approx 10.67
\]

\[
\frac{CF}{y} = \frac{24}{y} \quad \frac{36y}{32+y} = 24 \left( \frac{32+y}{3} \right) \quad \frac{36y}{256} + \frac{24y}{-24y} \quad \frac{12y}{12} = \frac{256}{12} \quad y = \frac{64}{3} \approx 21.33
\]

\[
\alpha_{\triangle ABC} = \frac{1}{2} \cdot bh = \frac{1}{2} \left( 16 + \frac{32}{3} + \frac{64}{3} \right)(36)
\]

\[
= 18 \left( 48 \right) = 864 \text{ sq units}
\]

\[
\frac{\alpha_{\triangle ABD}}{\alpha_{\triangle DEF}} = \left( \frac{BD}{EF} \right)^2 = \left( \frac{36}{24} \right)^2 = \left( \frac{3}{2} \right)^2 = \frac{9}{4}
\]

MORE ON BACK
2. (8 pts) Find the area. Assume that all sides meet at right angles.

\[ A_1 = 10(25) = 250 \text{ in}^2 \]
\[ A_2 = 9(19) = 171 \text{ in}^2 \]
\[ A_3 = 11(12) = 132 \text{ in}^2 \]

\[ A_{\text{total}} = 553 \text{ in}^2 \]

3. (4 pts) What did you learn this semester?