1. (8 pts) Given the figure shown below with $OS \parallel UR$, find the following:

   A. $OS$
   
   \[
   \frac{FS}{OS} = \frac{FR}{UR}
   \]
   
   \[
   OS = \frac{FS \cdot UR}{FR} = \frac{5(10)}{8} = \frac{50}{8} = \sqrt{625''} = 25''
   \]

   B. $OU$
   
   \[
   \frac{OU}{OF} = \frac{SE}{FS}
   \]
   
   \[
   \frac{y}{4} = \frac{3}{5} \quad y = \frac{3(4)}{5} = \frac{12}{5} = \sqrt{2.4''}
   \]

2. (6 pts) In $\triangle ABC$, if $AD = 24$ and $CD = 16$, find $BD$.

   \[
   \frac{BD}{CD} = \frac{AD}{AD}
   \]
   
   \[
   \frac{x}{16} = \frac{16}{24}
   \]
   
   \[
   \frac{24x}{24} = (16)^2 = \frac{256}{24}
   \]
   
   \[
   x = \frac{256}{24} \approx 10.67 = \frac{32}{3}
   \]
3. (8 pts) Given \( \triangle ABC \) shown below. Find the **exact** length of the missing side. Also find \( m\angle A \).

\[
\begin{align*}
X^2 + 5^2 &= 8^2 \\
X^2 + 25 &= 64 \\
X^2 &= 39 \\
X &= \sqrt{39} \text{ ft}
\end{align*}
\]

\[
\sin A = \frac{5}{8} = 0.625
\]

\[
m\angle A = \sin^{-1} \left( \frac{5}{8} \right) = 58.168^\circ
\]

4. (8 pts) Use the circle and secants to answer the following.

A. What is the measure of \( \angle AEC \)?

\[
m\angle AEC = \frac{1}{2} (100 + 20) = \frac{1}{2} (120) = 60^\circ
\]

\[
m\angle AEC = 180 - 60 = 120^\circ
\]

B. If \( AE = 4'' \), \( DE = 10'' \), and \( BE = 5'' \), find \( CE \).

\[
\begin{align*}
(AE)(DE) &= (BE)(CE) \\
CE &= \frac{(AE)(DE)}{BE} \\
&= \frac{4(10)}{5} = 8''
\end{align*}
\]

\[CE = 8''\]
5. (8 pts) An escalator is 508 feet long and the angle it forms with the horizontal is 32°. What is the vertical distance traveled if a passenger rides from the bottom to the top of the escalator? Round to the nearest tenth.

\[
\sin 32 = \frac{h}{508}
\]

\[
h = 508 \sin 32^\circ
\]

\[
h = 269.1989
\]

\[
h \approx 269.2\text{ ft}
\]

6. (8 pts) Suppose \( \triangle ABC \sim \triangle DEF \), \( AB = 5 \text{ cm} \), \( BC = 9 \text{ cm} \), and \( DE = 35 \text{ cm} \). Find \( EF \).

\[
\frac{AB}{DE} = \frac{BC}{EF}
\]

\[
EF = \frac{BC \cdot DE}{AC} = \frac{9 \cdot 35}{5} = 63 \text{ cm}
\]

7. (8 pts) Points A, B, and C are on circle O, as shown. \( AC = 18 \text{ inches} \) and \( m\angle AOB = 140^\circ \).

A. Find \( BC \).

\[
\sin A = \frac{BC}{AC}
\]

\[
BC = AC \cdot \sin A = 18 \sin (140^\circ) = 6.16\text{ in}
\]

B. Find \( m\angle OBC \).

\[
m\angle OBC = \frac{140^\circ}{2} = 70^\circ
\]
8. (16 pts) Given the figure shown below. Points E and F trisect diameter AC. Suppose AC = 18 inches.

Find:
A. Find the exact length of BC.

\[
\frac{AE}{BE} = \frac{BE}{CE} \Rightarrow \frac{BE}{CE} = \frac{6(\sqrt{2})}{6} \Rightarrow BC^2 = 2BE^2 = 72 \\
BC = \sqrt{72} = 6\sqrt{2} \text{ inches}
\]

B. Find the exact length of AB.

\[
AB^2 = AE^2 + BE^2 = 36 + 72 = 108 \\
AB = \sqrt{108} = 6\sqrt{3} \text{ inches}
\]

C. Find the exact ratio of \( \frac{BC}{AB} \).

\[
\frac{BC}{AB} = \frac{6\sqrt{2}}{6\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}
\]

D. Find the area of the rectangle ABCD to the nearest square inch.

\[
\text{Area} \ ABCD = (6\sqrt{3})(6\sqrt{2}) = 36 \sqrt{6} = 108\sqrt{2} \\
= 182.735 \text{ in}^2 \\
= 183 \text{ in}^2
\]
In \( \triangle ABC \), \( BD \perp AC \), \( EF \perp AC \), and \( AB \parallel DE \). \( BD = 40 \), \( AD = 16 \), and \( EF = 30 \).

Find the following:

\[
\frac{DF}{EF} = \frac{AD}{BD} = \frac{16}{40} = \frac{30 \times 16}{40} = 12
\]

\[
\frac{CF}{EF} = \frac{BF}{EF} = \frac{30^2}{12} = 75
\]

\[
\frac{AB}{DE} = \frac{30}{40} = \frac{4}{3} \approx 1.333
\]

Area of \( \triangle ABC \):

\[
\frac{1}{2} (AC)(BD) = \frac{1}{2} (16 + 12 + 75)(40) = \sqrt{2060}
\]

Area of \( \triangle ABD \):

\[
\frac{1}{2} (AD)(BD) = \frac{(16)(40)}{2} = \frac{(16)(40)}{(12)(30)} = \left( \frac{4}{3} \right)^2
\]

\[
\frac{1}{2} (DF)(EF) = \frac{16}{9} \approx 1.78
\]