Exam 3: Chapters 5 & 6
Math 90, Elementary Algebra, Section 7869
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100 points
Show all work to receive full credit. You may use a calculator. CHECK YOUR WORK!!!!

1. Simplify each expression. All numbers should be multiplied out!!!! Correct answers will contain only positive exponents. (3 pts each)

   A. \(-7^{-2}\)

   \[\frac{-1}{49}\]

   B. \(\left(2x^2\right)\left(5x^3\right)\left(x^9\right)\)

   \[\frac{10x^{14}}{}\]

   C. \(\left(\frac{x^3}{y^2}\right)^{-2}\left(x^{-1}y\right)^2\)

   \[\frac{x^6y^{-2}y^2}{x^2y^{-4}} = \frac{x^4y^2}{x^2y^{-4}} = x^2y^6\]

   D. \(\left(\frac{x^{-5}y^{-8}}{x^{-4}y^3}\right)^0\)

   \[1\]

   E. \(\left(\frac{x^{-7}}{x^{-9}}\right)^{-2}\)

   \[\frac{x^{14}}{x^{18}} = \frac{1}{x^4}\]

   F. \(\left(3x^{-2}y^8z^{-1}\right)^3\)

   \[\frac{3^3x^{-6}y^{24}z^{-3}}{} = \frac{27y^{24}}{x^6z^3}\]

2. Write the following numbers in scientific notation: (2 pts each)

   A. \(28,898,000,000\)

   \[2.8898 \times 10^{10}\]

   B. \(0.0000000824\)

   \[8.24 \times 10^{-8}\]
3. Perform the indicated operations. Combine all like terms and simplify each expression. (4 pts each)

A. \((12y^4 - 7y^3 + 2y^2) + (5y^4 + 3y^3 + 2y^2 + 1)\)

\[7y^4 - 4y^3 + 1\]

B. \((2x - 7y)^2\)

\[4x^2 - 28xy + 49y^2\]

C. \((a + 5b)(a - 5b)\)

\[a^2 - 25b^2\]

D. \(4x^3(-9x^3 + 6x^2y - 2xy^2 + 1)\)

\[-36x^6 + 24x^5y - 8x^4y^2 + 4x^3\]

E. \(\frac{y^3 - 3y^2 + 8y - 6}{y - 1}\) (Long Division)

\[\begin{array}{c|ccc}
- & y^2 & -2y & +6 \\
\hline
y-1 & y^3 & -3y^2 & +8y - 6 \\
& -y^3 & +y^2 & \\
\hline
& -2y^2 & +8y & +2y \\
& & -6y & +6 \\
& & & 6y - 6 \\
\end{array}\]

\[y^2 - 2y + 6\]

4. Factor each expression completely. If it cannot be factored, then write “PRIME”. (5 pts each)
A. $25y^2 - 64$
\[
(5y - 8)(5y + 8)
\]

B. $12r^2 + 18rq - 10r - 15q$
\[
6r(2r + 3q) - 5(2r + 3q)
\]
\[
= (2r + 3q)(6r - 5)
\]

C. $10a^2 - 17a + 3$
\[
M = 30 - 17 = 13
\]
\[
= \frac{-2 \pm \sqrt{(-1)^2 - 4\cdot10\cdot3}}{20}
\]
\[
= \frac{-2 \pm \sqrt{1 - 120}}{20}
\]
\[
= \frac{-2 \pm \sqrt{-119}}{20}
\]
\[
= \frac{-2 \pm 15}{20}
\]
\[
= \frac{-2 - 15}{20} \quad \text{or} \quad \frac{-2 + 15}{20}
\]
\[
= \frac{-17}{20} \quad \text{or} \quad \frac{13}{20}
\]
\[
= (5a - 1)(2a - 3)
\]

D. $x^2 + 36$
\[
\text{Prime}
\]

E. $b^4 - 81$
\[
= (b^2 - 9)(b^2 + 9)
\]
\[
= (b - 3)(b + 3)(b^2 + 9)
\]

F. $4x^2 - 28x + 49$
\[
= (2x - 7)^2
\]

G. $6m^4 + 3m^3 - 108m^2$
\[
= 3m^2(m^2 + m - 36)
\]
\[
= 3m^2(m - 6)(m + 6)
\]
\[
M = \frac{-B}{2A} = \frac{-4}{6} = -\frac{2}{3}
\]
\[
= \frac{9}{2m}
\]
\[
= \frac{3m^2(m - 4)(2m + 9)}{m - 9} + 1
\]
5. Solve each equation (using the zero factor property) (4 pts each).

A. \((4t+3)(t-2)=0\)
\[4t+3=0 \quad t-2=0\]
\[t=\frac{-3}{4} \quad t=2\]
\[\{t=\frac{-3}{4}, 2\}\]

B. \(n(n-5)=6\)
\[n^2-5n-6=0\]
\[(n-6)(n+1)=0\]
\[n=6, -1\]
\[\{n=-1, 6\}\]

6. (6 pts) The length of a rug is 6 ft more than its width. The total area of the rug is 40 ft\(^2\). Find the length and width of the rug. (HINT: DRAW A PICTURE)

\[\text{Width} = w \]
\[\text{Length} = w+6\]

\[w(w+6) = 40\]
\[w^2+6w-40=0\]
\[(w+10)(w-4)=0\]
\[w = -10, 4\]

\[\text{Width} = 4 \text{ ft}\]
\[\text{Length} = 10 \text{ ft}\]
7. The longer leg of a right triangle is 2 m longer than twice the shorter leg. The hypotenuse is 2 m shorter than three times the shorter leg.

A. (3 pts) What is the theorem you are to use in solving this problem? (write its equation).

PYTHAGOREAN Theorem

\[ a^2 + b^2 = c^2 \]

(HINT: Use this figure and label it appropriately)

B. (6 pts) Find the lengths of all three sides of the triangle. Circle your final answer.

\[
X^2 + (2X-2)^2 = (3X-2)^2
\]
\[
X^2 + 4X^2 - 8X + 4 = 9X^2 - 12X + 4
\]
\[
5X^2 - 8X - 4
\]
\[
\frac{5X^2 - 8X + 4}{4X^2 - 20X}
\]
\[
4X^2 - 20X = 0
\]
\[
4X(X - 5) = 0
\]
\[
X = 0, 5
\]

5 m, 12 m, 13 m

BONUS (total of 10 extra points)

The product of the smaller two of three consecutive integers is equal to 23 plus the largest. Find the integers.

\[
\text{LET } x = 157
\]
\[
x + 1 = 210
\]
\[
x + 2 = 212
\]

\[
x(x+1) = 23 + x+2
\]
\[
x^2 + x = 25 + x
\]
\[
-x - 25
\]
\[
-x - 25
\]

\[
x^2 - 25 = 0
\]
\[
(x - 5)(x + 5) = 0
\]
\[
x = 5, -5
\]

\[
\{5, 6, 7\}
\]
\[
\{5, -4, -3\}
\]