1. (4 pts each) Compute the following limits without using graphs or tables:

A. \[ \lim_{x \to 1} \frac{x-1}{3x^2-x-2} \]
\[
= \lim_{x \to 1} \frac{x-1}{(3x+2)(x-1)} = \frac{1}{5}
\]

B. \[ \lim_{x \to 0} \frac{8-x^2}{x^2} = \lim_{x \to 0} \frac{8-0}{0^2} = \frac{8-0}{0} = \frac{8}{0}
\]

C. \[ \lim_{h \to 0} \frac{10x^2h^2-5xh}{h} \]
\[
= \lim_{h \to 0} 10x^2h - 5x = -5x
\]

2. (14 pts) Consider the following piecewise function:

Graph \( f(x) \) by hand and find the following:

A. \[ \lim_{x \to 4^-} f(x) = -2 \]

B. \[ \lim_{x \to 4^+} f(x) = -2 \]

C. \[ \lim_{x \to 4} f(x) = -2 \]

D. \( f(4) = -2 \)

E. Is \( f(x) \) continuous? If it is not, then state the first condition from the definition of continuity that is violated.

Yes, \it{it is continuous}. 

\[ f(x) = \begin{cases} 
2x-10 & \text{if } x \geq 4 \\
2-x & \text{if } x < 4 
\end{cases} \]
3. (10 pts) Use the definition of the derivative, \( f(x) - \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \), to find \( f'(x) \) given \( f(x) = -2x^2 + 3x - 1 \). (SHOW EVERY STEP LEADING YOU TO YOUR ANSWER TO RECEIVE CREDIT.)

\[
f'(x) = \lim_{h \to 0} \frac{-2(x+h)^2 + 3(x+h) - 1 - (-2x^2 + 3x - 1)}{h}
\]

\[
= \lim_{h \to 0} \frac{-2(x^2 + 2xh + h^2) + 3x + 3h - 1 + 2x^2 - 3x + 1}{h}
\]

\[
= \lim_{h \to 0} \frac{-2x^2 - 4xh - 2h^2 + 3x + 3h - 1 + 2x^2 - 3x + 1}{h}
\]

\[
= \lim_{h \to 0} \frac{-4xh - 2h^2 + 3h}{h}
\]

\[
= \lim_{h \to 0} -4x - 2h + 3
\]

\[
= -4x + 3
\]

4. (10 pts total) A Company’s cost function is \( C(x) = 20 + 3x + \frac{54}{\sqrt{x}} \) dollars where \( x \) is the number of units (for \( 5 < x < 20 \)).

A. (4 pts) Find the Marginal Cost Function.

\[
MC(x) = C'(x) = 3 + \frac{54}{\sqrt{x}} \cdot \frac{1}{2} \cdot x^{-\frac{3}{2}}
\]

\[
MC(x) = \begin{cases} 
3 - 27x^{-\frac{3}{2}} & \text{if } x \neq 0 \\
\text{as } x \to 0 & \\
\text{as } x \to \infty & 
\end{cases}
\]

B. (4 pts) Find the Marginal Cost Function at \( x = 16 \).

\[
MC(16) = 3 - \frac{27}{\sqrt{16}} = 3 - \frac{27}{4} = 2.58
\]

C. (2 pts) Interpret your answer from part B.

WHEN 16 UNITS ARE PRODUCED, THE COST IS INCREASING AT THE RATE OF $2.58 PER UNIT PRODUCED.
5. (30 pts total) Find the derivative using the Product, Quotient, or Chain rules, then simplify whenever possible.

A. \( f(x) = \frac{x^4 + 1}{x^4 - 1} \)

\[
\frac{f'(x)}{f(x)} = \frac{4x^3(x^4 - 1) - 4x^3(x^4 + 1)}{(x^4 - 1)^2}
\]

\[
f'(x) = -\frac{8x^3}{(x^4 - 1)^2}
\]

B. \( f(x) = (x^2 + 4)(x^2 - 4) \)

\[
f'(x) = 2x(x^2 + 4) + 2x(x^2 - 4)
\]

\[
f'(x) = 4x^3
\]

C. \( f(x) = \frac{1}{\sqrt[3]{5x+1}} \)

\[
f'(x) = \frac{-3}{5} (5x + 1)^{-8/5}
\]

\[
f'(x) = -\frac{3}{5} (5x + 1)^{-8/5}
\]
6. (8 pts) Find \( f''(x) \) and \( f''(-2) \) given \( f(x) = \frac{-3}{4x^3} \).

\[
\begin{align*}
f'(x) &= -\frac{3}{4} (-3) x^{-4} = \frac{9}{4} x^{-4} \\
f''(x) &= \frac{9}{4} (-4) x^{-5} = -9 x^{-5} = \frac{-9}{x^5} \\
f''(-2) &= \frac{-9}{(-2)^5} = \frac{-9}{-32} = \frac{9}{32} = 0.28125
\end{align*}
\]

7. (16 pts total) Find the derivative using the appropriate rule.

A. \( f(t) = 4t^{3/2} - \frac{9}{3t} - \frac{3}{10}t^2 \)

\[
f'(t) = 4\left(\frac{3}{2}\right)t^{3/2} - 9\left(-\frac{1}{3}\right)t^{-4/3} - \frac{1}{5} \left(10t^2\right)^{-4/5}(2t)
\]

\[
f'(4) = -24t^{-3/2} + 3t^{-4/3} - 4t^2(10t^2)^{-4/5}
\]

B. \( g(x) = (2x+1)^3(2x-1)^4 \)

\[
g'(x) = 3(2x+1)^2(2)(2x-1)^4 + 4(2x-1)^3(2)(2x+1)^3
\]

\[
g'(x) = 6(2x+1)^2(2x-1)^4 + 8(2x-1)^3(2x+1)^3
\]
**BONUS (total of 15 extra points)**

A. (5 pts) Write the equation for the tangent line to the curve \( f(x) = 2x^2 + 3x + 1 \) at \( x = -2 \).

Write the equation in slope-intercept form.

\[
\begin{align*}
  y - y_1 &= m(x - x_1) \\
  y - 3 &= -5(x + 2) \\
  y &= -5x - 7
\end{align*}
\]

\[
m = f'(x) = 4x + 3 \\
f'(-2) = -8 + 3 = -5
\]

B. (10 pts) A rocket can rise to a height \( h(t) = t^2 + 0.5t^2 \) feet in \( t \) seconds.

i. Find its velocity 10 seconds after it is launched.

\[
\begin{align*}
  v(t) &= h'(t) = 3t^2 + 0.5(2t) = 3t^2 + t \\
  v(10) &= 3(10)^2 + 10 = \frac{310 \text{ FT}}{\text{SEC}}
\end{align*}
\]

ii. Find its acceleration 10 seconds after it is launched.

\[
\begin{align*}
  a(t) &= v'(t) = 6t + 1 \\
  a(10) &= 6(10) + 1 = \frac{61 \text{ FT}}{\text{SEC}^2}
\end{align*}
\]