Formula Sheet. Electricity and Magnetism,

Coulomb's law and electric fields

$$\vec{F}_{12} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12}$$

is the force on charge 1 due to charge 2. The unit vector points from charge 2 to charge 1.

$$\vec{F}(\text{on } q_0) = q_0\vec{E} \quad \text{where} \quad \vec{E} = \frac{kq}{r^2}\hat{r}$$

Principle of Superposition $\vec{E}_{net} = \sum_{i=1}^{n} \vec{E}_i$

Field from an infinitesimal charge element. $d\vec{E} = \frac{kdq}{r^2}\hat{r}$

Discontinuity at the surface of a charged plane $|\Delta E| = \frac{\sigma}{\varepsilon_0}$

Gauss's law

Flux: defined: $\phi_{net} = \oint_S \vec{E}\cdot\hat{n}dA = \oint_S E_n dA$

Gauss's Law: $\phi_{net} = 4\pi k q_{enclosed}$

$$= \frac{q_{enclosed}}{\varepsilon_0}$$

Electric potential

$$V(r) = \frac{kq}{r} \quad \text{and} \quad U = q_0 V$$

The above potential is the potential from a point charge as well as the potential outside a spherically symmetric charge distribution, with V=0 at infinity.

$dV = \frac{kdq}{r}$ potential from an infinitesimal charge element.

Potential calculated from the electric field

$$dV = -\vec{E}\cdot d\vec{\ell} \quad \text{and} \quad -\frac{dV}{d\ell} = E_{tan}$$

$$\Delta V = V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{\ell}$$

Constants

$e = 1.602 \times 10^{-19}$ C

$1 eV = 1.602 \times 10^{-19}$ J

$m_e = 9.11 \times 10^{-31}$ kg

$m_p = 1.67 \times 10^{-27}$ kg

$k = 1/(4\pi\varepsilon_o) = 8.99 \times 10^{9}$ N m^2/C^2

$\varepsilon_o = 8.85 \times 10^{-12}$ F/m (or C^2/N m^2)

$\mu_o = 4\pi \times 10^{-7}$ T m/A

$1\ T = 10^4$ G

Differential area element: dA

dA = (circumference)x(thickness)

ring: $dA = 2\pi r\, dr$

Differential volume elements: dv

dv = (surface area)x(thickness)

thin sheet: $dv = A\, dy$

thin cylindrical shell: $dv = 2\pi r L\, dr$

thin spherical shell: $dv = 4\pi r^2\, dr$

Capacitors

$$C = \frac{Q}{V} \qquad \text{parallel plate}: C = \frac{\varepsilon_0 A}{d}$$

parallel: $C_{equiv} = C_1 + C_2 + \ldots = \sum_{i=1}^{n} C_i$

series:

$$C_{equiv} = \frac{1}{1/C_1 + 1/C_2 + \ldots} = \frac{1}{\sum_{i=1}^{n} \frac{1}{C_i}}$$

Electrostatic Potential Energy

Point charges: $U = \sum_{i=1}^{n} q_i V_i$; bringing in each charge sequentially

Conductors at potential V: $U = \frac{1}{2}QV$

Energy in a capacitor:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

Energy density of electric fields

$$u_e = \frac{1}{2}\varepsilon_0 E^2$$

DC Circuits

$$I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} = qnAv_d$$

$$R = \rho\frac{L}{A} \qquad R = \frac{V}{I}$$

series: $R_{equiv} = R_1 + R_2 + \ldots = \sum_{i=1}^{n} R_i$

parallel:

$$R_{equiv} = \frac{1}{1/R_1 + 1/R_2 + \ldots} = \frac{1}{\sum_{i=1}^{n}\frac{1}{R_i}}$$

Ohm's Law

$V = IR$

Power dissipated in a resistor

$$P = IV = I^2R = \frac{V^2}{R}$$

Kirchhoff's rules

(1) at a junction: $\Sigma I_{in} = \Sigma I_{out}$

(2) around a closed loop: $\Sigma \Delta V = 0$

Time dependent circuits

charge on a capacitor:

derive I(t)and V(t) from Q(t)

RC, charging: $Q(t) = \mathcal{E}C\left(1 - e^{-t/\tau_{RC}}\right), \ \tau_{RC} = RC$

RC, discharging: $Q(t) = Q_o e^{-t/\tau_{RC}}$

current through an inductive circuit:

derive V(t) from I(t)

RL, close switch: $I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_{LR}}\right), \ \tau_{LR} = \frac{L}{R}$

RL, open switch: $I(t) = I_o e^{-t/\tau_{LR}}$

Magnetic Force

$$\vec{F}_B = q\vec{v}\times\vec{B} \quad [\,\vec{F}_{Lorentz} = q(\vec{E} + \vec{v}\times\vec{B})\,]$$

$$d\vec{F}_B = Id\vec{l}\times\vec{B} \qquad \vec{F} = I\vec{L}\times\vec{B}$$

Magnetic Torques on current loops

Magnetic moment: $\vec{\mu} = NIA\hat{n}$

$$\tau = NIAB\sin(\theta) \qquad \vec{\tau} = \vec{\mu}\times\vec{B}$$

potential energy of a current loop

$$U = -\vec{\mu}\cdot\vec{B}$$

Magnetic Fields

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \text{ (point charge : not in the book)}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2} \text{ Biot – Savart Law}$$

In the center of a single current loop

$$B_{loop} = \frac{\mu_o}{4\pi}\frac{2\pi I}{R}$$

On the axis of a single current loop

$$B_{loop}(x) = \frac{\mu_o}{4\pi}\frac{2\pi R^2 I}{\left(x^2 + R^2\right)^{3/2}} \quad \text{(x = 0 in the center)}$$

Inside a long solenoid

$B_x = \mu_0 nI$ (n is loops/m = N/L)

Due to a very long straight wire

$$B = \frac{\mu_0}{4\pi}\frac{2I}{R}$$

Due to a short straight segment

$$B = \frac{\mu_0}{4\pi}\frac{I}{R}(\sin\theta_2 - \sin\theta_1)$$

Ampere's Law

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_{encircled}$$

Induction

Flux in a single loop

$$\Phi_B = \int_S \vec{B}\cdot\hat{n}\,dA = BA\cos(\theta)$$

Faraday's law

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

Self inductance   Mutual inductance

$$L = \frac{N\Phi_B}{I} \qquad M = \frac{N_2\Phi_{21}}{I_1} = \frac{N_1\Phi_{12}}{I_2}$$

Self-induced (back) emf

$$\mathcal{E} = -L\frac{dI}{dt} \quad \text{or} \quad \mathcal{E} = -M\frac{dI}{dt}$$

Magnetic Energy

$U_m = \frac{1}{2}LI^2$ stored in an inductor

$u_m = \frac{1}{2}\frac{B^2}{\mu_0}$ energy density in a B field

Alternating Current

$$\varepsilon = \varepsilon_{peak} \sin(\omega t + \delta)$$

$$I_{rms} = \sqrt{(I^2)_{av}} = \frac{1}{\sqrt{2}} I_{peak}$$

resistor

$$I_{rms} = V_{R,rms} / R$$

inductor

$$I_{rms} = V_{L,rms} / \omega L = V_{L,rms} / X_L$$

capacitor

$$I_{rms} = V_{C,rms} / (1/\omega C) = V_{C,rms} / X_C$$

transformers

$$V_2 = {}^{N_2}\!/_{N_1} V_1$$

$$I_2 = {}^{N_1}\!/_{N_2} I_1$$