1. (16 pts) Consider the polynomial \( P(x) = x^4 - 2x^3 + 5x^2 - 8x + 4 \).
   a) List all possible rational zeros of \( P(x) \).
      \[ \pm 1, \pm 2, \pm 4 \]
   b) Find all rational and complex zeros of \( P(x) \). Use the quadratic formula if necessary.
      \[
      \begin{array}{c|cccc}
        & 1 & -2 & 5 & -8 & 4 \\
        \hline
        1 & 1 & -1 & 4 & -4 & 0 \\
      \end{array}
      \]
      \[
      \frac{x^2 - x^2 + 4x - 4}{x^2(x-1) + 4(x-1)} = 0
      \]
      \[
      (x-1)(x^2 + 4)
      \]
      \[
      x^2 = -4
      \]
      \[
      x = \pm 2i
      \]
      \[
      \text{Rational Zeros: } 1, \text{ Mult of 2}
      \]
      \[
      \text{Complex: } \pm 2i
      \]
   c) Write the complete factorization of \( P(x) \).
      \[
      P(x) = (x-1)^2 (x-2i)(x+2i)
      \]
   d) What is the remainder when \( P(x) \) is divided by \( x+3 \)? What is its significance?
      \[
      \begin{array}{c|cccc}
        -3 & 1 & -2 & 5 & -8 & 4 \\
        \hline
        0 & -3 & 15 & -60 & 208 \\
        4 & 0 & -3 & 20 & -68 & 208 \\
      \end{array}
      \]
      \[
      \text{Remainder } = 208
      \]
      \[
      P(-3) = 208
      \]
2. (22 pts) Consider the rational function \( R(x) = \frac{2x^2 - 6x - 7}{x^2 - 2x - 8} \).

a) Find the x- and y-intercepts of \( R(x) \).

- **X-intercept:** 
  \( \frac{x^2 - 6x - 7}{x^2 - 2x - 8} = 0 \)
  
  \[ x = 0 \rightarrow \frac{2x^2 - 6x - 7}{x^2 - 2x - 8} = 0 \]
  \[ \frac{3.90}{(3 - \sqrt{23}, 10)} \left( \frac{3 - \sqrt{23}, 10}{-0.90} \right) \]
  \[ y = 0 \rightarrow y = \frac{7}{8} \]
  \[ (0, \frac{7}{8}) \]

- **Y-intercept:** 
  \[ y = \frac{2x^2 - 6x - 7}{x^2 - 2x - 8} = 0 \]
  \[ y = \frac{7}{8} \]

b) Write an equation for each vertical and horizontal asymptote.

**VERTICAL ASYMPTOTE:**

\[ x = -2, \quad x = 4 \]

**HORIZONTAL ASYMPTOTE:**

\[ y = 2 \] when \( x \rightarrow -\infty \)

\[ x \rightarrow \infty \]

d) State the domain of \( R(x) \).

**DOMAIN:**

\[ \{ x \in \mathbb{R} \} \; \text{s.t.} \; x \neq -2, 4 \]

\[ (0, -2) \cup \left( -2, 4 \right) \cup (4, \infty) \]

d) Graph \( R(x) \) by hand, using the above information and by plotting points. Make sure to include all information from above (intercepts, asymptotes, and proper end behavior).
3. (9 pts) For each of the following rational functions, write an equation for every vertical, horizontal and/or slant asymptote. Only fill the boxes that apply, some boxes may remain empty!

<table>
<thead>
<tr>
<th></th>
<th>$h(x) = -\frac{3}{x}$</th>
<th>$h(x) = \frac{x+6}{x-2}$</th>
<th>$h(x) = \frac{x^2 + 3x + 2}{x+3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>$x = 0$</td>
<td>$x = 2$</td>
<td>$x = -3$</td>
</tr>
<tr>
<td>Horizontal</td>
<td>$y = 0$</td>
<td>$y = 1$</td>
<td></td>
</tr>
<tr>
<td>Slant</td>
<td></td>
<td></td>
<td>$y = x$</td>
</tr>
</tbody>
</table>

\[
\begin{array}{c|ccc}
-3 & 1 & 3 & 2 \\
\hline
0 & -3 & 0 & 1 \\
\hline
1 & 0 & 2 & 1 \\
\end{array}
\]

4. (8 pts) Below is the graph of $f(x) = 2^x$. On the same set of axes, graph $g(x) = -2^{x-3} + 3$ using transformations.

Describe the transformation in words.

- Shifted right 3 units,
- Up 3 units,
- Flipped upside down.

5. (9 pts) Simplify. Give an exact value for each. (NO DECIMAL approximations!!!)

a) $\ln\left(\frac{1}{e^3}\right)$
   
   \[ -3 \]

b) $-3 \log_8 512$
   
   \[ -3 = -3 \]

\[ -3 = \sqrt[3]{-27} \]

\[ x = \frac{2}{3} \]
6. (8 pts) A sum of $5000 is invested at an interest rate of 8 1/2 \% per year, compounded quarterly.
   a) Find the amount of the investment after 4 years.
   \[
   A(4) = 5000 \left(1 + \frac{0.085}{4}\right)^{(4)(4)} = 5000 \left(1.02125\right)^{16} = \$ 6999.76
   \]
   b) After what period of time will the investment total $12,000?
   \[
   \frac{12000}{5000} = \left(1 + \frac{0.085}{4}\right)^{4t}
   \]
   \[
   \frac{12}{5} = \left(1.02125\right)^{4t}
   \]
   \[
   \ln \left(\frac{12}{5}\right) = 4t \ln (1.02125)
   \]
   \[
   t = \frac{\ln (\frac{12}{5})}{4 \ln (1.02125)} = 10.41 \text{ years}
   \]

7. (12 pts) The half-life of palladium-110 is 4 days. After 20 days a sample has been reduced to a mass of 0.375 g.
   a) What was the initial mass of the sample?
   \[
   P(20) = 0.375 = P_0 e^{\ln (0.375)}
   \]
   \[
   \ln 0.375 = \ln P_0 + 5 \ln 0.5
   \]
   \[
   P_0 = e^{\ln 0.375 - 5 \ln 0.5} = 12 g
   \]
   b) Find a function that models the mass remaining after \( t \) days.
   \[
   P(t) = 12 e^{-0.173287 t}
   \]
   c) What is the mass after 3 days?
   \[
   P(3) = 12 e^{-0.173287(3)} = 12 e^{-0.519860} = 7.13 g
   \]
   d) After how many days will only 0.125 g remain?
   \[
   \frac{0.125}{12} = e^{-0.173287 t}
   \]
   \[
   \ln \left(\frac{0.125}{12}\right) = -0.173287 t
   \]
   \[
   t > 26.34 \text{ days}
   \]
   26 days 8 hrs 9.33 mins
8. (16 pts) Solve each equation for \( x \). Give your answer correct to two decimal places.

(a) \( \log_{x} 343 = 3 \)
\[
x^3 = 343
\]
\[
x = 7
\]

(b) \( \frac{5\ln(3-x)}{5} = \frac{4}{5} \)
\[
3-x = \exp\left(\frac{4}{5}\right) - 3
\]
\[
x = 3 - \exp\left(\frac{4}{5}\right)
\]
\[
x = 0.77
\]

(c) \( 10^{x+3} = 6^x \)
\[
(x+3)\ln 10 = 2\ln 6 \\
x\ln 10 + 3\ln 10 = 2\ln 6 \\
-3\ln 10 = -3\ln 10 \\
x\ln 10 - 2\ln 6 = -3\ln 10 \\
x = \frac{5139}{\ln 10 - 2\ln 6}
\]

(d) \( 2^{x+4} = \left(\frac{1}{8}\right)^x \)
\[
2^{x+4} = 2^{-3x} \\
x+4 = -3x \\
x = -2 \\
x = 2
\]

**BONUS (total of 10 extra points)**

**ANSWERS MUST BE EXACT!!!!!!**

A. Does \( f(x) = \frac{2x + 6}{x^2 + 4x + 3} \) have a vertical asymptote at \( x = -3 \)? EXPLAIN why or why not.
\[
f(x) = \frac{2(x+3)}{(x+1)(x+3)} = \frac{2}{x+1} \\
\text{But } x = -3 \text{ is not in the domain because of original eqn.}
\]
\[
\Rightarrow \text{No vertical asymptote, but hole in curve at } x = -3
\]

B. If \( 2^x = 7 \) what does \( 2^{-3x} = ? \)
\[
2^x = 7 \\
2^{-3x} = \left(2^x\right)^{-3} = 7^{-3} = \frac{1}{343}
\]