Section 12.10

Solving Probability Problems by Using Combinations

Example

A club consists of 5 men and 6 women. Four members are to be selected at random to form a committee. What is the probability that the committee will consist of two women?

\[
P\left(\text{two women}\right) = \frac{\text{# of possible committees with 2 woman}}{\text{total number of 4-member committees}}
\]

\[
= \frac{C_2^{\infty} \cdot C_4^{11}}{C_4^{15}} = \frac{\frac{6!}{2!4!}}{\frac{11!}{4!7!}} = \frac{\frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} \cdot \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1}}{\frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = \frac{15}{330} = \frac{1}{22}
\]
Example

- A bag contains four red balls and five green balls. You plan on selecting three balls at random. Determine the probability of selecting three green balls.

\[
P(3 \text{ green balls}) = \frac{\binom{5}{3}}{\binom{9}{3}}
\]

\[
= \frac{10}{84} = \frac{5}{42}
\]

Example

- You are dealt 5 cards from a standard deck of 52 cards. Determine the probability that you are dealt 5 red cards.

\[
P(5 \text{ red cards}) = \frac{\binom{26}{5}}{\binom{52}{5}}
\]

\[
= \frac{65780}{2598960} = \frac{253}{9969}
\]
Example

- The Honey Bear is testing 10 new flavors of ice cream. They are testing 5 vanilla based, 3 chocolate based and 2 strawberry based ice creams. If we assume that each of the 10 flavors has the same chance of being selected and that 4 new flavors will be produced, find the probability that
  a) no chocolate flavors are selected.
  b) at least 1 chocolate is selected.
  c) 2 vanilla and 2 chocolate are selected.

Solution

- 5 vanilla, 3 chocolate, 2 strawberry selecting 4 flavors

  a) \[ P(\text{no chocolate}) = \frac{7}{10} \binom{4}{4} = \frac{35}{210} = \frac{1}{6} \]

  b) \[ P(\text{at least 1 chocolate}) = 1 - P(\text{no chocolate}) \]
      \[ = 1 - \frac{1}{6} = \frac{5}{6} \]
Solution (continued)

- 5 vanilla, 3 chocolate, 2 strawberry; selecting 4 flavors

\[ P(2 \text{ vanilla and } 2 \text{ chocolate}) = \frac{\binom{5}{2} \cdot \binom{3}{2}}{\binom{10}{4}} = \frac{10 \cdot 3}{210} = \frac{30}{210} = \frac{1}{7} \]

Example

- An airline is given permission to fly 4 new routes of its choice. The airline is considering 12 new routes: 4 routes in FL, 5 routes in CA, and 3 routes in TX. If the airline selects the 4 new routes at random from the 12 possibilities, determine the probability that
  - 2 are in FL and 2 are in TX
  - 3 are in CA and 1 is in FL
  - 1 is in FL, 1 is in CA, and 2 are in TX
  - At least one is in TX
Example

An airline is given permission to fly 4 new routes of its choice. The airline is considering 12 new routes: 4 routes in FL, 5 routes in CA, and 3 routes in TX. If the airline selects the 4 new routes at random from the 12 possibilities, determine the probability that

- 2 are in FL and 2 are in TX  \( \frac{2}{55} \)
- 3 are in CA and 1 is in FL  \( \frac{8}{99} \)
- 1 is in FL, 1 is in CA, and 2 are in TX
- At least one is in TX
Example

An airline is given permission to fly 4 new routes of its choice. The airline is considering 12 new routes: 4 routes in FL, 5 routes in CA, and 3 routes in TX. If the airline selects the 4 new routes at random from the 12 possibilities, determine the probability that

- 2 are in FL and 2 are in TX \( \frac{2}{55} \)
- 3 are in CA and 1 is in FL \( \frac{8}{99} \)
- 1 is in FL, 1 is in CA, and 2 are in TX \( \frac{4}{33} \)
- At least one is in TX \( \frac{41}{55} \)