Chapter 12

Probability

WHAT YOU WILL LEARN

• Empirical probability and theoretical probability
• Compound probability, conditional probability, and binomial probability
• Odds against an event and odds in favor of an event
• Expected value
• Tree diagrams
WHAT YOU WILL LEARN

• Mutually exclusive events and independent events
• The counting principle, permutations, and combinations

Section 1

The Nature of Probability
Definitions

- An **experiment** is a controlled operation that yields a set of results.
- The possible results of an experiment are called its **outcomes**.
- An **event** is a subcollection of the outcomes of an experiment.

Definitions (continued)

- **Empirical probability** is the relative frequency of occurrence of an event and is determined by actual observations of an experiment.
- **Theoretical probability** is determined through a study of the possible outcomes that can occur for the given experiment.
Empirical Probability

\[ P(E) = \frac{\text{number of times event } E \text{ has occurred}}{\text{total number of times the experiment has been performed}}\]

- **Example:** In 100 tosses of a fair die, 19 landed showing a 3. Find the empirical probability of the die landing showing a 3.
- Let \( E \) be the event of the die landing showing a 3.
\[ P(E) = \frac{19}{100} = 0.19 \]

The Law of Large Numbers

- The **law of large numbers** states that probability statements apply in practice to a large number of trials, not to a single trial. It is the relative frequency over the long run that is accurately predictable, not individual events or precise totals.
Section 2

Theoretical Probability

Equally likely outcomes

- If each outcome of an experiment has the same chance of occurring as any other outcome, they are said to be **equally likely outcomes**.
- For equally likely outcomes, the probability of Event $E$ may be calculated with the following formula.

\[ P(E) = \frac{\text{number of outcomes favorable to } E}{\text{total number of possible outcomes}} \]
Example

- A die is rolled. Find the probability of rolling
  - a) a 2.
  - b) an odd number.
  - c) a number less than 4.
  - d) an 8.
  - e) a number less than 9.

Solutions: There are six equally likely outcomes: 1, 2, 3, 4, 5, and 6.

- a) \[ P(2) = \frac{\text{number of outcomes that will result in a 2}}{\text{total number of possible outcomes}} = \frac{1}{6} \]

- b) There are three ways an odd number can occur: 1, 3 or 5.
  \[ P(\text{odd}) = \frac{3}{6} = \frac{1}{2} \]

- c) Three numbers are less than 4.
  \[ P(\text{number less than 4}) = \frac{3}{6} = \frac{1}{2} \]
Solutions: There are six equally likely outcomes: 1, 2, 3, 4, 5, and 6 (continued)

- d) There are no outcomes that will result in an 8.

\[ P(\text{number greater than 8}) = \frac{0}{6} = 0 \]

- e) All outcomes are less than 9. The event must occur and the probability is 1.

Important Facts

- The probability of an event that cannot occur is 0.
- The probability of an event that must occur is 1.
- Every probability is a number between 0 and 1 inclusive; that is, \( 0 \leq P(E) \leq 1 \).
- The sum of the probabilities of all possible outcomes of an experiment is 1.
Example

A standard deck of cards is well shuffled. Find the probability that the card is selected.

a) a 10.
b) not a 10.
c) a heart.
d) an ace, 2, or 3.
e) diamond and spade.
f) a card greater than 4 and less than 7.

Example (continued)

a) a 10

There are four 10’s in a deck of 52 cards.

\[ P(10) = \frac{4}{52} = \frac{1}{13} \]

b) not a 10

\[ P(\text{not a 10}) = 1 - P(10) = 1 - \frac{1}{13} = \frac{12}{13} \]
Example continued

c) a heart

There are 13 hearts in the deck.

\[ P(\text{heart}) = \frac{13}{52} = \frac{1}{4} \]

d) an ace, 2 or 3

There are 4 aces, 4 twos and 4 threes, or a total of 12 cards.

\[ P(A, \ 2, \ \text{or} \ 3) = \frac{12}{52} = \frac{3}{13} \]

Example continued

d) diamond and spade

The word \textit{and} means both events must occur. This is not possible.

\[ P(\text{diamond and spade}) = \frac{0}{52} = 0 \]

e) a card greater than 4 and less than 7

The cards greater than 4 and less than 7 are 5's and 6's (or a total of 8 cards).

\[ P(>\ 4 \ \text{and} \ <\ 7) = \]

\[ P(5 \ \text{or} \ 6) = \frac{8}{52} = \frac{2}{13} \]