Math 180

Chapter 6 Review Solutions

1. a) \[ \int_{-4}^{4} \left[ 20 - x^2 - (x^2 - 12) \right] dx = \frac{512}{3} \]

b) \[ \int_{0}^{2} \left[ e^{x-1} - (x^2 - x) \right] dx = e^2 - \frac{11}{3} \]

2. a) \[ \int_{-2}^{2} (8 - x^3) dx = \frac{31}{4} \]

b) shells. \[ \int_{0}^{2\pi} x(8 - x^3) dx = \frac{38\pi}{5} \]

3. a) disks: \[ r = x^3 \]

\[ \int_{0}^{1} \pi (x^3)^2 dx = \frac{\pi}{7} \]
b) Washer

\[ R = 5 \]
\[ r = y^2 + 1 \]

\[ \int_0^2 \pi (5^2 - (y^2 + 1)^2) dy = \frac{544\pi}{15} \]

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\[ \int_0^4 2\pi x \left(\frac{4}{x}\right) dx = 24\pi \]

\[ \int_1^4 2\pi x \left(\frac{4}{x}\right) dx = 24\pi \]

\[ \int_0^3 2\pi (x+2)(3x-x^2) dx = \frac{63\pi}{2} \]

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\[ \int_{-2}^2 2\pi (3-y)(4-y^2) dy = 64\pi \]

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4. a) 
\[ \int_0^4 \pi (8-x^{3/2})^2 \, dx = \frac{576\pi}{5} \]

b) need to find \( k \in [0,4] \) such that \( \int_0^k \pi (8-x^{3/2})^2 \, dx \) is \( \frac{1}{2} \) of \( \frac{576\pi}{5} = 361.91 \)

\[ \int_0^k \pi (8-x^{3/2})^2 \, dx = \pi \int_0^k (64-16x^{3/2}+x^3) \, dx \]

\[ = \pi \left( 64x - \frac{32}{5} x^{5/2} + \frac{x^4}{4} \right) \Bigg|_0^k = \pi \left( 64k - \frac{32}{5} k^{5/2} + \frac{k^4}{4} \right) \]

So set \( = \frac{288\pi}{5} \), and get all terms to one side of equation.

\[ \pi (64k - \frac{32}{5} k^{5/2} + \frac{k^4}{4}) - \frac{288\pi}{5} = 0 \]

Use calculator to find zero on \([0,4]: \) \( k = 0.999904 \)

5. \( F(x) = kx \)
\( F(9) = k(9) = 20 \)
\( k = \frac{20}{9} \)

\( W = \int_0^1 \frac{20}{9} x \, dx = \frac{10}{9} \) J

6. \( W = 18 = \int_0^4 kx \, dx = \frac{kx^2}{2} \Bigg|_0^4 = \frac{k(4)^2}{2} - 0 \)

\( 18 = \frac{0.08}{k} \)
\( k = 225 \)
\( \) So \( F(x) = 225x \)

\( W = \int_0^5 225x \, dx = 10,125 \) J

7. Work = \( \int_0^{20} 5x \, dx = \frac{5x^2}{2} \Bigg|_0^{20} = \frac{5(20)^2}{2} = 1000 \) ft-lbs
8. Work to lift cat only = \( F \cdot d = (5 \text{ lbs})(18 \text{ ft}) = 90 \text{ ft-lbs} \)
   Work to lift rope only = \( \int_0^{18} 1x \, dx = \frac{x^2}{2} \bigg|_0^{18} = \frac{18^2}{2} = 162 \text{ ft-lbs} \)
   Total work done = 90 + 162 = \textbf{252 ft-lbs}

9. \[
\frac{f_{ave}}{4-1} = \frac{1}{4-1} \int_1^4 (3x^2 - 2x) \, dx = 16
\]

10. \[
\text{Average population} = \frac{1}{10-0} \int_0^{10} 5.2e^{\sqrt{x}} \, dx \quad \text{need calculator and fnInt}
\]
   \[
   = \frac{1}{10} (\text{fn Int}(5.2e^{\sqrt{x}}, x, 0, 10))
   \]
   \[
   = 54.2 \text{ thousand people}
   \]
   or 54,200 people