1. Evaluate the following definite and indefinite integrals (if they exist).

   a. \( \int_{1}^{3} (x^2 + 3x) \, dx \)

   b. \( \int_{0}^{1} (3 - x)^5 \, dx \)

   c. \( \int_{1}^{4} \sqrt{x} (x - 2) \, dx \)

   d. \( \int_{0}^{1} \frac{dx}{(3x - 1)^2} \)

   e. \( \int_{0}^{\sqrt{\pi}} x \sin(x^2) \, dx \)

   f. \( \int_{0}^{e} \frac{4}{2x - 1} \, dx \)

   g. \( \int_{1}^{(x^3 + 1)\sqrt{x^4 + 4x}} dx \)

   h. \( \int_{0}^{1} x^4 \sec^2(x^5) \, dx \)

   i. \( \int_{0}^{1/2} \frac{1}{4x^2} \, dx \)

   j. \( \int_{0}^{\sqrt{1-x^2}} \frac{2}{\sqrt{1-x^2}} \, dx \)

   k. \( \int_{0}^{\pi} \frac{\sin x}{\cos x} \, dx \)

   l. \( \int_{0}^{\pi} \cos x \cdot e^{\sin x} \, dx \)

   m. \( \int_{0}^{e} \frac{x^2 + x + 1}{x} \, dx \)

   n. \( \int_{1}^{e} \frac{\ln x}{x} \, dx \)
2. a. Evaluate the Riemann sum for \( f(x) = x^2 - x \) \( 0 \leq x \leq 2 \) with five subintervals, taking the sample points to be
   i. right endpoints
   ii. left endpoints

   b. Consider \( \int_0^2 (x^2 - x) \, dx \). Draw a diagram to explain the geometric meaning of this integral.

   c. Use the Fundamental Theorem of Calculus to find \( \int_0^2 (x^2 - x) \, dx \) exactly.

3. Find the exact velocity function given the following acceleration function and initial velocity.
   \( a(t) = t - 3, \quad v(0) = 4 \)

4. Use 5 rectangles to approximate the area under the given graph of \( f \) from \( x = 0 \) to \( x = 5 \)
   a. Use left endpoint approximations.
   b. Use right endpoint approximations.
   c. Use midpoint approximations.

5. What is the difference between the types of answers obtained by computing \( \int f(x) \, dx \) and \( \int_a^b f(x) \, dx \)?

6. If \( f \) is a differentiable function and \( 0 < a < b \), then what does \( \frac{d}{dx} \left[ \int_a^b f(x) \, dx \right] \) equal?
7. A bottle of wine at room temperature (68°F) is placed in a refrigerator at 4 pm. Its temperature after $t$ hr is changing at the rate of $-18e^{-0.6t}$ degrees Fahrenheit/hour.

Explain in context what the following integral represents $\int_{0}^{3} -18e^{-0.6t} \, dt$ and compute the definite integral.

8. An oil storage tank ruptures at time $t = 0$ and oil leaks from the tank at a rate $r(t) = 100e^{-0.01t}$ of liters per minute. How much oil leaks out during the first hour?

9. A particle moves along a line with velocity function $v(t) = t^2 - t$, where $v$ is measured in meters per second. Find (a) the displacement and (b) the total distance traveled by the particle during the time interval [0, 5].

10. A car is moving along a straight road from A to B, starting at A at time $t = 0$. Below is a graph of the cars velocity $v(t)$, measured in miles per hour.

a. How many miles away from A is the car at time $t = 6$?

b. At what time does the car change direction?

c. What does $\int_{0}^{8} v(t) \, dt$ represent in this problem?

d. What was the TOTAL distance the car traveled in the 8 hours?
11. The graph of the function $f$, consisting of three line segments, is given.

Let \( g(x) = \int_{-2}^{x} f(t) dt \)

a. Compute \( g(-2) \), \( g(0) \), \( g(1) \) and \( g(4) \).

b. Sketch the graph of \( g(x) \) on the same grid to the right →

c. Find the instantaneous rate of change of \( g \), with respect to \( x \), at \( x = 3 \).
   In other words: \( g'(3) \)

d. Find the absolute maximum value of \( g \) on the closed interval \([-2, 4]\). Justify your answer.

12. Let the function \( F \) be defined by \( F(x) = \int_{0}^{x} \frac{t^2 - 2t}{e^t} \ dt \).

a. Find \( F'(x) \) and \( F''(x) \).

b. If they exist, determine the critical points for \( F \).

c. Discuss the concavity of the graph of \( F \).

13. The following figure shows the graphs of \( f \), \( f' \), and \( \int_{a}^{x} f(t) dt \). Identify each graph, and explain your choices.
1. a. \( \frac{62}{3} \)  
   b. \( \frac{665}{6} \)  
   c. \( \frac{46}{15} \)  
   
   d. Can’t do since we can’t use FTC. \( \frac{1}{(3x-1)^2} \) is not continuous at \( x=1/3 \)  
   
   e. \( \frac{1}{2} - \frac{\sqrt{2}}{4} \)  
   f. \( 2 \ln|2x-1| + C \)  
   g. \( \frac{1}{6}(x^4 + 4x)^{3/2} + C \)  
   
   h. \( \frac{1}{5} \tan(x^2) + C \)  
   i. \( -\frac{1}{4x} + C \)  
   j. \( \frac{\pi}{3} \)  
   k. \( -\ln|\cos x| + C \)  
   
   l. \( e^{ax} + C \)  
   m. \( \frac{e^2}{2} + e - \frac{1}{2} \)  
   n. \( \frac{1}{2} \)  

2. a. i. \( R_5 = 1.12 \)  
   ii. \( L_5 = 0.32 \)  

3. \( v(t) = \frac{1}{2}t^2 - 3t + 4 \)  

4. (These are approximate from the graph, your answers may differ slightly) 
   a. \( I_5 \approx 25.6 \)  
   b. \( R_5 \approx 19.6 \)  
   c. \( M_5 \approx 23.8 \)  

5. \( \int_a^b f(x)dx \) is an indefinite integral and gives a family of function, the antiderivatives of \( f(x) \).
\( \int_a^b f(x)dx \) is a definite integral from \( a \) to \( b \) and gives a number, which can be interpreted as net area between the curve \( y = f(x) \) and the x-axis.  

6. \( \frac{d}{dx} \left[ \int_a^b f(x)dx \right] = 0 \), since \( \int_a^b f(x)dx \) is a number, and the derivative of a constant is zero.  

7. The integral represents the net change in the temperature of the bottle of wine over the 3 hour time period from 4pm to 7pm in degrees Fahrenheit. That is, it is the number of °F the temperature of the bottle fell in 3 hours while being refrigerated. We expect this value to be negative, as the temperature is falling as the bottle is cooled. \( \int_0^3 -18e^{-0.6t} \, dt = -8.35°F \)
8. \[ \int_0^6 100e^{0.6t} \, dt \approx 4511.9 \text{ liters} \]

9. a. Displacement = \( \frac{175}{6} \approx 29.2 \text{ meters} \)  
   b. Total distance traveled = 29.5 meters

10. a. 9 miles  
   b. \( t = 5 \text{ hours} \)  
   c. Net distance or displacement from A after 8 hours.  
   d. 16 miles

11. a. Interpret the integral as net area under the curve from -2 to \( x \), so  
   \[ g(-2) = 0 \quad g(0) = 4 \quad g(1) = 6 \quad g(4) = 5 \]
   b.  
   c. \( g'(3) = f(3) = -2 \) by FTC- Part 1.
   d. Looking at graph of \( g(x) \) from part (b), The absolute maximum value is 7, and is found at \( x = 2 \).

12. a. Find \( F''(x) = \frac{x^2 - 2x}{e^x} \), and \( F''(x) = \frac{-x^2 + 4x - 2}{e^x} \)
   b. critical values are \( x = 0 \) and \( x = 2 \)  
   (local max at \( x = 0 \), local min at \( x = 2 \))
   c. Concave up \( (2 - \sqrt{2}, 2 + \sqrt{2}) \), Concave down \( (-\infty, 2 - \sqrt{2}) \cup (2 + \sqrt{2}, \infty) \)

13. First note that either a or b must be the graph of \( \int_0^1 f(t) \, dt \), since \( \int_0^1 f(t) \, dt = 0 \) and \( c(0) \neq 0 \). 
Now notice that b > 0 when c is increasing, and that c > 0 when a is increasing. It follows that c is the graph of \( f(x) \), b is the graph of \( f'(x) \), and a is the graph of \( \int_0^1 f(t) \, dt \).