1. a. $f'(x) = 3x^2 + 3x - 36 = 0$, Critical values: $x = -4, 3$
   Increasing: $(-\infty, -4) \cup (3, \infty)$
   Decreasing: $(-4, 3)$
   b. Local Max = 111 @ $x = -4$, Local Min = -60.5 @ $x = 3$
   c. $f'(x) = 6x + 3 = 0$, Critical value: $x = -\frac{1}{2}$
   Concave up: $(-1/2, \infty)$
   Concave down: $(-\infty, -1/2)$
   d. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
   $x_1 = 0$
   $x_2 = 0.19444444$
   $x_3 = 0.19625914$
   $x_4 = 0.19625933$
   $x_5 = 0.19625933$
   The root is 0.19625933.
   e. The root is 0.19625933.

2. C.V.'s: Local MIN @ $x = 0$, Local MIN @ $x = -\frac{3 - \sqrt{5}}{2}$, Local MAX @ $x = -\frac{3 + \sqrt{5}}{2}$

3. a. Max value of $\frac{1}{4}$ at $x = 2$, Min value of 0 at $x = 0$.
   b. Max value of $2\pi$ at $\theta = 2\pi$, Min value of 0 at $\theta = 0$.

4. intercept (0,0)
   H.A. $y = 0$: as $x \to \infty$, $f(x) \to 0$, but as $x \to -\infty$, $f(x) \to \infty$
   Max $(2, \frac{4}{e^2})$ Min $(0, 0)$ Inflection points $x = 2 \pm \sqrt{2}$, which are marked below.
5. Set \( y'' = 0 \), to find inflection point at \((2, -16)\), \( y'(2) = -12 = \text{slope of tangent line} \)
So line tangent to \( y \) at \((2, -16)\) is \( y = -12x + 8 \)

6. \( f'(x) = \frac{1}{x} \), so \( f'(1) = 1 \) and the linear approximation is \( L(x) = x - 1 \).
\( \ln(0.98) \approx L(0.98) = 0.98 - 1 = -0.02 \).

7. Since \( f(x) \) is differentiable on \([-1, 2]\), it is also continuous on \([-1, 2]\), thus \( f \) satisfies the conditions of the Mean Value Theorem. Therefore, there exists \( c \) in the interval \((-1, 2)\), such that
\[
\frac{f(2) - f(-1)}{2 - (-1)} = \frac{5 - (-1)}{3} = \frac{6}{3} = 2.
\]
This means that at \( x = c \), the slope of the line tangent is 2.
Which is the same slope of the line \( y=2x \)

8. See figure labeled with the unknown lengths →
The constraint equation: \( xy = 30 \) which means \( y = \frac{30}{x} \).
The objective function to be minimize is:
\[
A = (x + 2)(y + 4) \quad \text{with the substitution: } \quad y = \frac{30}{x}.
\]
\[
A = (x + 2)\left(\frac{30}{x} + 4\right) = 30 + 4x + \frac{60}{x} + 8
\]
\[
A = 38 + 4x + \frac{60}{x}
\]
And the derivative: \( A' = 4 - \frac{60}{x^2} = 0 \) gives the solution, \( x = \sqrt{15} \) and \( y = \frac{30}{\sqrt{15}} \).
So the dimensions of the page are \( \left(\sqrt{15} + 2\right) \text{ inches} \) by \( \left(\frac{30}{\sqrt{15}} + 4\right) \text{ inches} \),
giving a minimum area of 69.98 square inches.

9. a. 0  b. 10.5  c. \( e^0 = 1 \)

10. Constraint: \( P = 1500 = 3x + y \implies y = 1500 - 3x \)
Objective fctn: \( A = xy = x(1500 - 3x) \)
Set \( A'(x) = 0 \) and solve….. \( x = 250 \text{ft}, y = 750 \text{ft} \)

11. 

Graphical representation...

12. \( f(x) \) has a point of inflection at \((0, f(0))\). \( f''(x) \) is negative for \( x<0 \), and positive for \( x>0 \).

13. We are finding the root of \( x^3 - 2 = 0 \), so \( f(x) = x^3 - 2 \)

In calculator: \( y1 = x^3 - 2 \), \( y2 = f'(x) = 3x^2 \)

In calculator: \( 1 \rightarrow x \) ENTER, \( x - y_1 / y_2 \rightarrow x \) ENTER, ENTER, …. 

\( x_1 = 1 \)
\( x_4 = 1.2599 \)

\( x_2 = 1.3333 \)  \( x_3 = 1.2599 \) This is the value of \( \sqrt[3]{2} \) correct to 4 decimals.

\( x_3 = 1.2639 \)

14. a. \( x_1 = -1 \) is a “bad seed” in that the function happens to have a local minimum at \( x = -1 \).

Thus Newton’s method does not work since the horizontal tangent does not produce an \( x \)-intercept.

In other words, since \( f'(1) = 0 \), you cannot compute \( x_2 \), since you cannot divide by 0 in the calculation: \( x_2 = -1 - \frac{f(-1)}{f'(-1)} \).

b. We are finding the root of \( x^3 - 3x + 6 = 0 \), so \( f(x) = x^3 - 3x + 6 \)

In calculator: \( y1 = x^3 - 3x + 6 \), \( y2 = f'(x) = 3x^2 - 3 \)

In calculator: \( -2 \rightarrow x \) ENTER, \( x - y_1 / y_2 \rightarrow x \) ENTER, ENTER, …. 

\( x_1 = -2 \)
\( x_4 = -2.355309 \)

\( x_2 = -2.444444 \)  \( x_3 = -2.355301 \) This is the root correct to 6 decimals.

\( x_3 = -2.359158 \)

15. a. \( dy = \frac{1}{2\sqrt{x}} \) \( dx \)  

b. \( dy = \frac{1}{2\sqrt{25}}(0.03) = 0.003 \)

c. \( \sqrt{25.03} = \sqrt{25} + \Delta y \approx 5 + dy = 5 + 0.003 = 5.003 \), so \( \sqrt{25.03} \approx 5.003 \)

16. a. \( f(x) \) is continuous and differentiable on \([-1, 2]\).

b. Set \( f'(x) = \frac{f(2) - f(-1)}{2 - (-1)} \rightarrow \frac{2}{(x + 2)^2} = \frac{1}{2} \)

Solve for \( x \): \( x = 0, -4 \)  \( c = 0 \) is in the interval \([-1, 2]\).