1. Differentiate the functions:
   a. \( f(x) = x^4 - \frac{2}{3x^2} \)
      \[ f'(x) = 4x^3 + \frac{4}{3x^3} \]
   b. \( y = 4\pi x^2 \)
      \[ y' = 0 \]
   c. \( h(x) = \frac{2x - 4}{x^3 + 2x + 1} \)
      \[ h'(x) = \frac{-4x^3 + 12x^2 + 10}{(x^3 + 2x + 1)^2} \]
   d. \( F(x) = \sin x \cdot (1 + \cos x) \)
      \[ F'(x) = \cos x + \cos^2 x - \sin^2 x \]
   e. \( s(x) = \sqrt{1 - \tan x} \)
      \[ s'(x) = -\sec^2 x \cdot \frac{1}{2\sqrt{1 - \tan x}} \]
   f. \( y = \ln(\cos(x^2)) \)
      \[ y' = -2x \tan(x^2) \]
   g. \( y = 5\sec(3x) \)
      \[ y' = 15\sec(3x)\tan(3x) \]
   h. \( n(x) = \pi^x \)
      \[ n'(x) = \pi^x \ln \pi \]
   i. \( y = xe^{\cos x} \)
      \[ y' = e^{\cos x} (1 - x \sin x) \]
   j. \( h(x) = \frac{3x - 4}{5x + 1} \)
      \[ h'(x) = \frac{23}{(5x + 1)^2} \]
   k. \( f(x) = \ln(x^6 + 1) \)
      \[ f'(x) = \frac{6x^5}{x^6 + 1} \]
   l. \( y = (3x)^x \) use logarithmic diff!
      \[ y' = (3x)^x (\ln(3x) + 1) \]
   m. \( g(x) = \csc(4x) \)
      \[ g'(x) = -4\csc(4x)\cot(4x) \]
   n. \( y = \sin^{-1}(6x) \)
      \[ y' = \frac{6}{\sqrt{1 - 36x^2}} \]

2. Let \( h(x) = \sqrt{1 - x^2} \cdot \arcsin(x) \). Find \( \frac{dh}{dx} \) and simplify where possible.

   \[ \frac{dh}{dx} = 1 - \frac{x\sin^{-1}(x)}{\sqrt{1 - x^2}} \]

3. Find an equation of a tangent line to \( y^2 = x^3(2 - y) \) at the point \((1, 1)\). Put your answer in the form \( y = mx + b \)

   Differentiate implicitly: \[ \frac{dy}{dx} = \frac{3x^2(2 - y)}{2y + x^3}, \text{ so } m = 1 \]

   Tangent line at \((1, 1)\): \( y = x \) See graph to verify.
4. At what point on the curve \( y = \left[ \ln(x + 4) \right]^2 \) is the tangent line horizontal?

Need \( y' = 0 \). So solve \( y' = \frac{2\ln(x + 4)}{x + 4} = 0 \) \( \rightarrow \) when \( x = -3 \)

So the tangent line is horizontal at \((-3, 0)\).

See graph to verify.

5. Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 \( \text{cm}^3/\text{s} \). How fast is the radius of the balloon increasing when the diameter is 50 cm?

Note: The volume of a sphere is \( V = \frac{4}{3} \pi r^3 \)

\[
\frac{dr}{dt} = \frac{1}{25\pi} \text{cm/s} \approx 0.0127 \text{cm/s}
\]

6. Find the equation of the tangent line to \( y = \ln(e^x + e^{2x}) \) at the point \((0, \ln 2)\).

Slope = \( y'(x) = \frac{e^x + 2e^{2x}}{e^x + e^{2x}} \) at \( x = 0 \), \( y'(0) = \frac{3}{2} \)

So equation of tangent line is \( y = \frac{3}{2}x + \ln 2 \).

See graph to verify.

7. Let \( H(x) = f(g(x)) \). Find \( H'(2) \)

By chain rule: \( H'(x) = f'(g(x))g'(x) \), so \( H'(2) = f'(g(2))g'(2) \)

From the graph \( g(2) = -2, g'(2) = -1 \), so we have \( H'(2) = (f'(-2)) \cdot -1 \)
\( f'(-2) = 2 \), so that gives \( H'(2) = (2) \cdot -1 = -2 \).

\( H'(2) = -2 \)
8. Two people start from the same point. One walks east at 6 mi/h and the other walks northeast at 4 mi/h. How fast is the distance between the people changing after 30 minutes?

Use law of cosines.
\[ c^2 = a^2 + b^2 - 2ab \cos \left( \frac{\pi}{4} \right) \]

30 minutes = \( \frac{1}{2} \) hour
a = 2, b = 3, \( c = 2.12479 \), \( \frac{da}{dt} = 4 \), \( \frac{db}{dt} = 6 \)
\[ c^2 = a^2 + b^2 - 2ab \cos \left( \frac{\pi}{4} \right) \]

\[ 2c \frac{dc}{dt} = 2a \frac{da}{dt} + 2b \frac{db}{dt} - \sqrt{2} \left( a \frac{db}{dt} + b \frac{da}{dt} \right) \]
Plug and chug and solve for \( \frac{dc}{dt} = 4.24957 \) mph

9. A particle moves on a vertical line so that its coordinate at time \( t \) is \( s(t) = t^3 - 12t + 3 \), \( t \geq 0 \).
   a. Find the velocity and acceleration functions.
   \[ v(t) = s'(t) = 3t^2 - 12 \]
   \[ a(t) = s''(t) = 6t \]

   b. When is the particle moving upward and when is it moving downward?
   The particle moving upward when \( v(t) = 3t^2 - 12 = 3(t - 2)(t + 2) > 0 \).
   That happens when \( t > 2 \) or \( t < -2 \). In this problem, only \( t > 2 \) makes sense.
   The particle moving downward when \( v(t) = 3t^2 - 12 < 0 \).
   That happens when \(-2 < t < 2 \). Only \( 0 < t < 2 \) makes sense in this problem.

c. When is the particle speeding up and when is it slowing down?
   The particle is speeding up when the sign of \( v(t) \) is the same as the sign of \( a(t) \).
   That happens when \( -2 < t < 0 \) or \( t > 2 \). Only \( t > 2 \) makes sense in this problem.
   The particle is slowing down when the signs are different.
   That happens when \( t < -2 \) or \( 0 < t < 2 \). Only \( 0 < t < 2 \) makes sense in this problem.

10. \( f = c \), the zero of “c” does not correspond to a max or min (0 slope) in the other 2 functions.
    \( f' = a \), the zero of “a” corresponds the minimum (0 slope) of “c”.
    \( f'' = b \), the zero of “b” corresponds to the maximum (0 slope) of “a”.
11.  
   a. t = 6 sec, when v(t) switches from positive to negative, the bug will stop crawling UP the wire, and start crawling DOWN the wire.
   
   b. 0 < t < 6 sec
   
   c. 6 < t < 9 sec
   
   d. speed = |v(t)|, is greatest when 2 ≤ t ≤ 5 and when 7 ≤ t ≤ 8. The bug is crawling up (or down) the wire at 6 mm/sec during those times.
   
   e. t = 0 sec, 6 sec, 9 sec. This is when v(t) = 0.
   
   f. No. The bug is crawling up the wire for 6 seconds, and only crawling down the wire for 4 seconds. The bug will not return to the bottom within the first 9 seconds.