

$$1) \boxed{f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}}$$

$$\begin{aligned}
 2) \quad f'(x) &= \lim_{h \rightarrow 0} \frac{[2(x+h)^2 + 5(x+h) - 1] - [2x^2 + 5x - 1]}{h} \\
 &= \lim_{h \rightarrow 0} \frac{[2x^2 + 4xh + 2h^2 + 5x + 5h - 1] - [2x^2 + 5x - 1]}{h} \\
 &= \lim_{h \rightarrow 0} \frac{4xh + 2h^2 + 5h}{h} = \lim_{h \rightarrow 0} \frac{h(4x + 2h + 5)}{h} \\
 &= \lim_{h \rightarrow 0} 4x + 2h + 5 = \boxed{4x + 5}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad f(t) &= 2t^{-\frac{1}{2}} + 4t^{-\frac{3}{2}} + 2 \\
 f'(t) &= 2\left(-\frac{1}{2}t^{-\frac{3}{2}}\right) + 4\left(-\frac{3}{2}t^{-\frac{5}{2}}\right) = -t^{-\frac{3}{2}} - 6t^{-\frac{5}{2}} \\
 &= \boxed{\frac{-1}{t^{\frac{3}{2}}} - \frac{6}{t^{\frac{5}{2}}}}
 \end{aligned}$$

$$\begin{aligned}
 4) \quad g(s) &= 2s^2 - \frac{4}{s} + \frac{2}{\sqrt{s}} = 2s^2 - 4s^{-1} + 2s^{-\frac{1}{2}} \\
 g'(s) &= 4s + 4s^{-2} - s^{-\frac{3}{2}} = \boxed{4s + \frac{4}{s^2} - \frac{1}{s^{\frac{3}{2}}}}
 \end{aligned}$$

$$\begin{aligned}
 5) \quad h(x) &= (2x^2 + x)^3 \\
 h'(x) &= 3(2x^2 + x)^2 \underbrace{(4x + 1)}_{\text{chain}} = \boxed{(12x + 3)(2x^2 + x)^2}
 \end{aligned}$$

6) $g(t) = \frac{t}{t^2+4}$ Find $g''(t)$... use quotient rule.

$$g'(t) = \frac{(t^2+4)(1) - t(2t)}{(t^2+4)^2} = \frac{t^2+4 - 2t^2}{(t^2+4)^2}$$

$$= \frac{-t^2+4}{(t^2+4)^2}$$

$$g''(t) = \frac{(t^2+4)^2(-2t) - (-t^2+4) \left[2(t^2+4) \frac{(2t)}{\text{chain}} \right]}{(t^2+4)^4}$$

$$= \frac{\cancel{(t^2+4)}(2t) \left[-1 - (-t^2+4)(2) \right]}{(t^2+4)^3}$$

$$= \frac{2t(-1+2t^2-8)}{(t^2+4)^3} = \boxed{\frac{2t(2t^2-9)}{(t^2+4)^3}}$$

7) $f(x) = x(x^2+1)^3$ (Product Rule)

$$f'(x) = x'(x^2+1)^3 + x(x^2+1)^3'$$

$$= (x^2+1)^3 + x(3(x^2+1)^2(2x))$$

$$= (x^2+1)^3 + 6x^2(x^2+1)^2$$

$$= (x^2+1)^2 \left[(x^2+1) + 6x^2 \right]$$

$$= (x^2+1)^2 \left[7x^2+1 \right]$$

$$f''(x) = \left(2(x^2+1)(2x) \right) (7x^2+1) + (x^2+1)^2 (14x)$$

$$= 2x(x^2+1) \left[2(7x^2+1) + 7x(x^2+1) \right]$$

$$= \boxed{2x(x^2+1) (7x^3+14x^2+7x+2)}$$

8.) $f(t) = 46.9(1 + 1.09t)^{0.1}$
 $t = 0$ Corresponds to 1900

$\therefore t = 100$ Corresponds to 2000.

$$f(100) = 46.9(1 + 1.09(100))^{0.1}$$

$$= \boxed{75.0433 \text{ years}}$$

Rate of change: $f'(t) = 46.9(0.1)(1 + 1.09t)^{-0.99} (1.09)$
chain

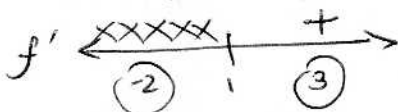
$$f'(100) = (46.9)(0.1)(1.09)(1 + 1.09(100))^{-0.99}$$

$$= \boxed{.04871 \text{ years/year}}$$

9) $f(x) = \sqrt{x-1} = (x-1)^{1/2}$

$$f'(x) = \frac{1}{2}(x-1)^{-1/2} = \frac{1}{2\sqrt{x-1}}$$

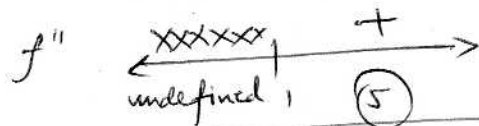
Critical point @ $x = 1$.



↑
Undefined

- a) Increasing on $(1, \infty)$
 b) Min @ $x = 1$.

$$f''(x) = -\frac{1}{4}(x-1)^{-3/2} = \frac{-1}{4(x-1)^{3/2}}$$



- c) Concave \uparrow on $(1, \infty)$
 d) No inflection points.

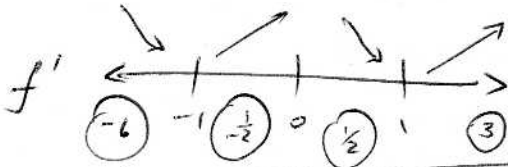
$$10) f(x) = x^4 - 2x^2$$

$$f'(x) = 4x^3 - 4x = 0$$

$$\rightarrow 4x(x^2 - 1) = 0$$

$$\rightarrow 4x(x+1)(x-1) = 0$$

$$\rightarrow \text{C.P.} : x = 0, -1, 1$$

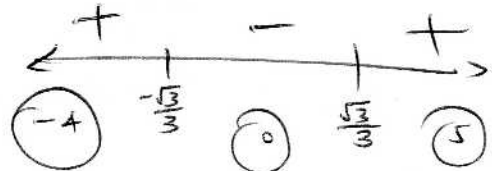


- a) Increasing on $(-1, 0) \cup (1, \infty)$
 Decreasing on $(-\infty, -1) \cup (0, 1)$
 b) Max @ $x = 0$
 Min @ $x = -1, 1$

$$f''(x) = 12x^2 - 4 = 0$$

$$\rightarrow 4(3x^2 - 1) = 0$$

$$\text{C.P.'s} \rightarrow x = \pm \sqrt{\frac{1}{3}} = \pm \frac{\sqrt{3}}{3}$$



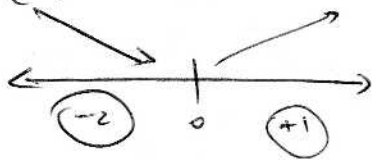
- c) Concave \uparrow on $(-\infty, -\frac{\sqrt{3}}{3}) \cup (\frac{\sqrt{3}}{3}, \infty)$
 Concave \downarrow on $(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$
 d) I.P. @ $x = \pm \frac{\sqrt{3}}{3}$

$$11) f(x) = -(1+x^2)^{-1}$$

$$f'(x) = (1+x^2)^{-2} (2x)$$

$$= \frac{2x}{(1+x^2)^2}$$

$$\text{C.P.} : x = 0$$



- a) Inc @ $(0, \infty)$
 Dec @ $(-\infty, 0)$
 b) Min @ $x = 0$

$$f''(x) = 2(1+x^2)^{-2} + 2x(-2)(1+x^2)^{-3}(2x)$$

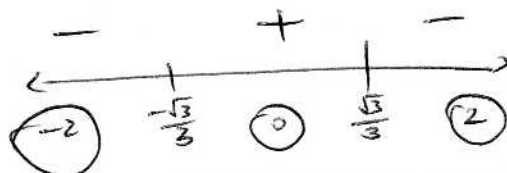
Product Rule

$$= 2(1+x^2)^{-3} [(1+x^2) + 2x^2(-2)]$$

$$= 2(1+x^2)^{-3} (-3x^2 + 1)$$

$$= \frac{2(1-3x^2)}{(1+x^2)^3}$$

$$\text{C.P.'s} : 1 - 3x^2 = 0 \rightarrow x = \pm \sqrt{\frac{1}{3}} = \pm \frac{\sqrt{3}}{3}$$



- c) C \uparrow @ $(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$
 C \downarrow @ $(-\infty, -\frac{\sqrt{3}}{3}) \cup (\frac{\sqrt{3}}{3}, \infty)$
 d) I.P.'s @ $x = \pm \frac{\sqrt{3}}{3}$

