1) $\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$

2) $$f'(x) = \lim_{h \to 0} \frac{[2(x+h)^2 + 5(x+h) - 1] - [2x^2 + 5x - 1]}{h}$$

$$= \lim_{h \to 0} \frac{[\cancel{2x^2} + 4xh + 2h^2 + \cancel{5x} + 5h - \cancel{1}] - [\cancel{2x^2} + \cancel{5x} - \cancel{1}]}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2 + 5h}{h} = \lim_{h \to 0} \frac{\cancel{h}(4x + 2h + 5)}{\cancel{h}}$$

$$= \lim_{h \to 0} 4x + \cancel{2h}^{\to 0} + 5 = \boxed{4x + 5}$$

3) $$f(t) = 2t^{-1/2} + 4t^{-3/2} + 2$$

$$f'(t) = 2\left(-\tfrac{1}{2}t^{-3/2}\right) + 4\left(-\tfrac{3}{2}t^{-5/2}\right) = -t^{-3/2} - 6t^{-5/2}$$

$$= \boxed{\frac{-1}{t^{3/2}} - \frac{6}{t^{5/2}}}$$

4) $$g(s) = 2s^2 - \frac{4}{s} + \frac{2}{\sqrt{s}} = 2s^2 - 4s^{-1} + 2s^{-1/2}$$

$$g'(s) = 4s + 4s^{-2} - s^{-3/2} = \boxed{4s + \frac{4}{s^2} - \frac{1}{s^{3/2}}}$$

5) $$h(x) = (2x^2 + x)^3$$

$$h'(x) = 3(2x^2+x)^2 \underbrace{(4x+1)}_{\text{Chain}} = \boxed{(12x+3)(2x^2+x)^2}$$

①

6) $g(t) = \frac{t}{t^2+4}$ Find $g''(t)$ ... use quotient rule.

$$g'(t) = \frac{(t^2+4)(1) - t(2t)}{(t^2+4)^2} = \frac{t^2+4-2t^2}{(t^2+4)^2}$$

$$= \frac{-t^2+4}{(t^2+4)^2}$$

$$g''(t) = \frac{(t^2+4)^2(-2t) - (-t^2+4)\left[2(t^2+4)\underbrace{(2t)}_{\text{chain}}\right]}{(t^2+4)^4}$$

$$= \frac{\cancel{(t^2+4)}(2t)\left[-1 - (-t^2+4)(2)\right]}{(t^2+4)^{\cancel{4}\,3}}$$

$$= \frac{2t\left(-1+2t^2-8\right)}{(t^2+4)^3} = \boxed{\frac{2t(2t^2-9)}{(t^2+4)^3}}$$

7) $f(x) = x(x^2+1)^3$ (PRODUCT RULE)

$$f'(x) = x'(x^2+1)^3 + x(x^2+1)^{3\,\prime}$$

$$= (x^2+1)^3 + x\left(3(x^2+1)^2(2x)\right)$$

$$= (x^2+1)^3 + 6x^2(x^2+1)^2$$

$$= (x^2+1)^2\left[(x^2+1) + 6x^2\right]$$

$$= (x^2+1)^2\left[7x^2+1\right]$$

$$f''(x) = \left(2(x^2+1)(2x)\right)\left(7x^2+1\right) + (x^2+1)^2(14x)$$

$$= 2x(x^2+1)\left[2(7x^2+1) + 7x(x^2+1)\right]$$

$$= \boxed{2x(x^2+1)\left(7x^3+14x^2+7x+2\right)}$$

8.) $f(t) = 46.9(1+1.09t)^{0.1}$

$t=0$ Corresponds to 1900

$\therefore$ $t=100$ Corresponds to 2000.

$f(100) = 46.9\left(1+1.09(100)\right)^{0.1}$

$= \boxed{75.0433 \text{ years}}$

Rate of Change: $f'(t) = 46.9\left((0.1)(1+1.09t)^{-0.99}\underbrace{(1.09)}_{\text{Chain}}\right)$

$f'(100) = (46.9)(0.1)(1.09)\left(1+1.09(100)\right)^{-0.99}$

$= \boxed{.04871 \text{ years}/\text{year}}$

9) $f(x) = \sqrt{x-1} = (x-1)^{1/2}$

$f'(x) = \frac{1}{2}(x-1)^{-1/2} = \frac{1}{2\sqrt{x-1}}$

Critical point @ $x=1$.

f': XXXXX | + ; (-2) 1 (3)

Undefined

a) Increasing on $(1,\infty)$.

b) Min @ $x=1$.

$f''(x) = -\frac{1}{4}(x-1)^{-3/2} = \frac{-1}{4(x-1)^{3/2}}$

f'': XXXXXX | + ; undefined 1 (5)

c) Concave ↑ on $(1,\infty)$

d) No inflection points.

10) $f(x) = x^4 - 2x^2$

$f'(x) = 4x^3 - 4x \;:= 0$

$\longrightarrow 4x(x^2-1) = 0$

$\longrightarrow 4x(x+1)(x-1) = 0$

$\longrightarrow$ C.P. : $x = 0, -1, 1$.

f'

$-6 \quad -1 \quad -\frac{1}{2} \quad 0 \quad \frac{1}{2} \quad 1 \quad 3$

a) Increasing on $(-1,0) \cup (1,\infty)$
Decreasing on $(-\infty,-1) \cup (0,1)$

b) Max @ $x = 0$
Min @ $x = -1, 1$.

$f''(x) = 12x^2 - 4 \;:= 0$

$\longrightarrow 4(3x^2 - 1) = 0$

$\xrightarrow{\text{C.P.'s}}$ $x = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{3}}{3}$

$+ \quad - \quad +$

$-4 \quad -\frac{\sqrt{3}}{3} \quad 0 \quad \frac{\sqrt{3}}{3} \quad 5$

c) Concave $\uparrow$ on $(-\infty, -\frac{\sqrt{3}}{3}) \cup (\frac{\sqrt{3}}{3}, \infty)$
Concave $\downarrow$ on $(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$

d) I.P. @ $x = \pm\frac{\sqrt{3}}{3}$.

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11) $f(x) = -(1+x^2)^{-1}$

$f'(x) = (1+x^2)^{-2}(2x)$

$= \dfrac{2x}{(1+x^2)^2}$

C.P. : $x = 0$

$-2 \quad 0 \quad +1$

a) Inc @ $(0,\infty)$
Dec @ $(-\infty,0)$

b) Min @ $x = 0$

$f''(x) = \underbrace{2(1+x^2)^{-2} + 2x\left(-2(1+x^2)^{-3}\underbrace{(2x)}_{\text{chain}}\right)}_{\text{Product Rule}}$

$= 2(1+x^2)^{-3}\left[(1+x^2) + 2x^2(-2)\right]$

$= 2(1+x^2)^{-3}(-3x^2+1)$

$= \dfrac{2(1-3x^2)}{(1+x^2)^{+3}}$

C.P.'s : $1 - 3x^2 = 0 \longrightarrow x = \pm\sqrt{\frac{1}{3}} = \frac{\pm\sqrt{3}}{3}$

$- \quad + \quad -$

$-2 \quad -\frac{\sqrt{3}}{3} \quad 0 \quad \frac{\sqrt{3}}{3} \quad 2$

c) C$\uparrow$ @ $(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$
C$\downarrow$ @ $(-\infty, -\frac{\sqrt{3}}{3}) \cup (\frac{\sqrt{3}}{3}, \infty)$

d) IP's @ $x = \pm\frac{\sqrt{3}}{3}$.