M178 Exam 2 – Fall 2004

Name: _____

Instructions: To receive credit, you must show all work leading to an answer. Circle or box your final answer.

1. Find the **absolute extreme values** of the function $f(x) = \dfrac{1}{x^2+1}$ on $[-4,\ 4]$.

2. If the amount of a drug in a person's blood after t hours is given by $f(t) = \dfrac{t}{t^2+4}$, when will the drug concentration be the greatest? (**maximum** concentration?).

3. Use implicit differentiation to find $\dfrac{dy}{dx}$ if $x^2 = (y-2)^3 + 1$.

4. Find $\dfrac{dy}{dx}$ and evaluate at the given values

   (a) $x^5 - y^3 = 1$ when $x = 1$ and $y = 1$.

   (b) $2x^3 - 5xy = 14$ when $x = 1$ and $y = 0$.

5. For the function $f(x) = \frac{3}{4}x^4 - 2x^3 + 2$

   (a) Make a sign diagram for $f'(x)$

   (b) Make a sign diagram for $f''(x)$

   (c) Sketch the graph by hand. **Label all relative extreme points. Label all inflection points**.

6. A computer store can sell 6 computers per day at a price of \$2000. Tom, the owner, estimates that for each \$100 price reduction he can sell 2 more computers per day. If each computer costs him \$1200 with 500 fixed costs, answer the questions below.
   Write down the following functions before solving the problem.

   (a) The price function $p(x) =$

   (b) The price function $q(x) =$

(c) The price function $R(x) =$

(d) The price function $C(x) =$

(d) The profit function $P(x) =$

Find the **price** that will maximize profit and find the **number that will be sold** at that price. Also find the **maximum profit.**

(note: x = **the number of decreases** not the number of computers YOU DO NOT NEED TO GET A WHOLE NUMBER FOR X so long as your quantity is a whole number)

7. A wine warehouse expects to sell 30,000 bottles of wine in a year. Each bottle costs the store \$9, plus a fixed charge of \$200 per order. If it costs \$3 to store a bottle for a year, how many bottles should be ordered at a time and how many orders should the warehouse place in a year to minimize inventory costs?

   Write down the following functions before solving:

Storage	costs =
Reordering	costs =
Total	costs =

   How many bottles should be ordered at a time and how many orders should be placed in a year to minimize inventory costs?

8. A company's profit from selling x units of an item is $P(x) = 2000x - \frac{3}{2}x^2$ dollars. If sales are growing at a rate of 30 per day, find how rapidly profit is growing (in dollars per day) when 500 units have been sold.

   (You must find $\dfrac{dP}{dt}$)

9. An athletic shoe company finds that it costs \$20 to produce each pair of shoes and that fixed costs are \$100 per day. The price function is $p(x) = 620 - 5x$, p is the price in \$ at which exactly x pairs of shoes will be sold. (keep in mind that x must be a whole number).

   Find the quantity (x) that should be produced and the price that should be charged to maximize profit.

10. Suppose that the relationship between the tax rate t on imported wine and the total sales S in millions of dollars is given by $S(t) = 30 - 60\sqrt[3]{t}$. Find the tax rate t that MAXIMIZES **revenue for the government**.